NAG Toolbox for MATLAB

f08pa

1 Purpose

f08pa computes the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z for an n by n real nonsymmetric matrix A.

2 Syntax

```
[a, sdim, wr, wi, vs, info] = f08pa(jobvs, sort, select, a, 'n', n)
```

3 Description

The real Schur factorization of A is given by

$$A = ZTZ^{\mathrm{T}},$$

where Z is orthogonal, the matrix of Schur vectors, and T is upper quasi-triangular with 1 by 1 and 2 by 2 diagonal blocks.

A matrix is in real Schur form if it is upper quasi-triangular with *1* by *1* and *2* by *2* blocks. *2* by *2* blocks will be standardized in the form

$$\begin{bmatrix} a & b \\ c & a \end{bmatrix}$$

where $bc < 0$. The eigenvalues of such a block are $a \pm \sqrt{bc}$.

Optionally, f08pa also orders the eigenvalues on the diagonal of the real Schur form so that selected eigenvalues are at the top left. The leading columns of Z form an orthonormal basis for the invariant subspace corresponding to the selected eigenvalues.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobvs – string**

If **jobvs** = 'N', Schur vectors are not computed.

If **jobvs** = 'V', Schur vectors are computed.

Constraint: **jobvs** = 'N' or 'V'.

2: **sort – string**

Specifies whether or not to order the eigenvalues on the diagonal of the Schur form.

sort = 'N'

Eigenvalues are not ordered.

sort = 'S'

Eigenvalues are ordered (see user-supplied logical function **select**).

Constraint: **sort** = 'N' or 'S'.

3: **select – string containing name of m-file**

If **sort** = 'S', **select** is used to select eigenvalues to sort to the top left of the Schur form.

If **sort** = 'N', **select** is not referenced and f08pa may be called with the string `'f08paz'`.

Its specification is:

```
[result] = select(wr, wi)
```

Input Parameters

1: **wr – double scalar**

2: **wi – double scalar**

An eigenvalue $\mathbf{wr}(j) + \sqrt{-1} \times \mathbf{wi}(j)$ is selected if $\mathbf{select}(\mathbf{wr}(j), \mathbf{wi}(j))$ is **true**. If either one of a complex conjugate pair of eigenvalues is selected, then both are. Note that a selected complex eigenvalue may no longer satisfy $\mathbf{select}(\mathbf{wr}(j), \mathbf{wi}(j)) = \mathbf{true}$ after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case **info** is set to $\mathbf{n} + 2$ (see **info** below).

Output Parameters

1: **result – logical scalar**

The result of the function.

4: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n matrix A.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldvs, work, lwork, bwork

5.4 Output Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

a contains its real Schur form T.

2: **sdim – int32 scalar**

If **sort** = 'N', **sdim** = 0.

If **sort** = 'S', **sdim** = number of eigenvalues (after sorting) for which user-supplied logical function **select** is **true**. (Complex conjugate pairs for which **select** is **true** for either eigenvalue count as 2.)

3: **wr**$(*)$ **– double array**

Note: the dimension of the array **wr** must be at least $\max(1, \mathbf{n})$.

See the description of **wi**.

4: **wi**$(*)$ **– double array**

Note: the dimension of the array **wi** must be at least $\max(1, \mathbf{n})$.

wr and **wi** contain the real and imaginary parts, respectively, of the computed eigenvalues in the same order that they appear on the diagonal of the output Schur form T. Complex conjugate pairs of eigenvalues will appear consecutively with the eigenvalue having the positive imaginary part first.

5: **vs**(**ldvs**,$*$) **– double array**

The first dimension, **ldvs**, of the array **vs** must satisfy

if **jobvs** = 'V', **ldvs** $\geq \max(1, \mathbf{n})$;
ldvs ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$ if **jobvs** = 'V', and at least 1 otherwise

If **jobvs** = 'V', **vs** contains the orthogonal matrix Z of Schur vectors.

If **jobvs** = 'N', **vs** is not referenced.

6: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobvs**, 2: **sort**, 3: **select**, 4: **n**, 5: **a**, 6: **lda**, 7: **sdim**, 8: **wr**, 9: **wi**, 10: **vs**, 11: **ldvs**, 12: **work**, 13: **lwork**, 14: **bwork**, 15: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info = 1 to N

If **info** $= i$ and $i \leq \mathbf{n}$, the QR algorithm failed to compute all the eigenvalues

info $= N + 1$

The eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned).

info $= N + 2$

After reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy **select** $=$ **true**. This could also be caused by underflow due to scaling.

7 Accuracy

The computed Schur factorization satisfies

$$A + E = ZTZ^{\mathrm{T}},$$

where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.8 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3.

The complex analogue of this function is f08pn.

9 Example

f08pa_select.m

```
function [result] = select(wr, wi)
  if (wi == 0)
    result = true;
  else
    result = false;
  end
```

```
jobvs = 'Vectors (Schur)';
sort = 'Sort';
a = [0.35, 0.45, -0.14, -0.17;
     0.09, 0.07000000000000001, -0.54, 0.35;
     -0.44, -0.33, -0.03, 0.17;
     0.25, -0.32, -0.13, 0.11];
[aOut, sdim, wr, wi, vs, info] = f08pa(jobvs, sort, 'f08pa_select', a)
```

```
aOut =
    0.7995   -0.0059   -0.0751   -0.0927
         0   -0.1007    0.3937    0.3569
         0         0   -0.0994   -0.5128
         0         0    0.3132   -0.0994
sdim =
          2
wr =
    0.7995
   -0.1007
   -0.0994
   -0.0994
wi =
         0
         0
    0.4008
   -0.4008
vs =
   -0.6551   -0.1210   -0.5032    0.5504
   -0.5236   -0.3286    0.7857    0.0229
```

```
    0.5362   -0.5974    0.0904    0.5894
   -0.0956   -0.7215   -0.3482   -0.5908
info =
           0
```